Problem 1: express the function  as a sum of sine waves, each with period 2L. Let L = 40 cm. Use excel to graph your sum keeping only the first 5 terms, and make a second graph summing the first 50 terms. See last paragraph of this handout for hints.

The following summary may be useful in doing this homework problems. It’s a review of what we did in class to express a sine wave as a sum of cosine waves.

For a wave on a string, we want both ends fixed - i.e., have y(x) = 0 at x=0 and x=L, where L is the length of the string.. Normally, that would lead us to describe a wave on a string in terms of sine waves. However, since our claim is that we can make ANY “even” wave out of cosines, in class I decided to show how to make a sine out of cosines.

Here’s the wave we want to describe: A string is L = 60 cm long, and the amplitude of the wave is 1 cm. The function is 

If we want to describe this ONLY with cosines, we need the function to be “even.” So for x= L to 0, we define . Then the function that we’ll analyze looks like this:

This function is even, with “period” P = 120 cm. We assume it would repeat indefinitely like this.

Because it is a repeating function, Fourier assures us we can express is as a sum of sines and cosines. If we take the “repeating” function to be from -60 cm to + 60 cm (P = 120 cm), then the function is even. That will prove useful in getting rid of the sine terms.

In general, we know (from Monday’s class) that

If g(x) is any function that REPEATS with period P, then to arbitrary “accuracy,”



where  or 

and  or .,

and we make the useful definition . Since P = 2L, that’s . .

First let’s find the Bm. . We can break this integral into 2 parts, one from –L to 0, the other from 0 to +L.. So:



You should be able to convince yourself that the first and second integrals cancel each other out. In fact, for ANY even function, all values of Bm are zero. That, of course, was why we extended the original f(x) as an even function – so that the only contributions would be cosine terms.

Now the Am : . We again break the integral into 2 parts and find:



You should be able to convince yourself that here, the first and second integrals are equal, so



or equivalently, . And in fact, ANY function f(x) defined from 0 to L can be expressed as a sum of cosine waves with . But remember that **these waves have wavelength 2L, not L**.

So 

This integral is easy to do if you express the sine and cosine as complex terms; maybe Michael will do that with you if he hasn’t already. In class I suggested you google “wolfram definite integrator” if you do so, put in

sin(pi\*x/L)\*cos(m\*pi\*x/L)dx and integrate from 0 to L. The computer will tell you that the integral is . Since , we get

. Those were the values I put into the spreadsheet. When I do so and add up to m=7, I get:

and when I add up to m=20, I get a very good approximation to that sine wave:

Along with this handout you should be getting 2 spreadsheets. These repeat sums I showed in class. One shows a square wave of period 2π. The second shows the sine wave I’ve just discussed. To play with the first sheet (square wave), you should only change the contents of the 2 cells in green: the period and the number of terms you’re adding. The rows “An” and “Bn” are the results of the integration. The rows labeled “partial\_An” and “partial\_Bn” are the values of the coefficients UP TO the value of “n” you put into cell B5. Incidentally, “partial\_Ao” includes the 1/2 required for term 0.

The second sheet (sine wave) contains the results derived above. **Modify it** to solve the problem at the beginning of this sheet. Print your curves, and e-mail me the modified spreadsheet.